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Scalar Gravity and Higgs Mechanism

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The role that the auxiliary scalar field ϕ plays in Brans–Dicke cosmology is discussed. If a constant vacuum energy is assumed to be the origin of dark energy, then the corresponding density parameter would be a quantity varying with ϕ; and almost all of the fundamental components of our universe can be unified into the dynamical equation for ϕ. As a generalization of Brans–Dicke theory, we propose a new gravity theory with a complex scalar field φ which is coupled to the cosmological curvature scalar. Through such a coupling, the Higgs mechanism is naturally incorporated into the evolution of the universe, and a running density of the field vacuum energy is obtained which may release the particle standard model from the rigorous cosmological constant problem in some sense. Our model predicts a running mass scale of the fundamental particles in which the gauge symmetry breaks spontaneously. The running speed of the mass scale in our case could survive all existing experiments.

KEY WORDS: cosmology; Higgs mechanism; vacuum energy.

1. INTRODUCTION

Recent observational data, in particular the Hubble diagram of type I supernova (Perlmutter *et al.*, 1998) and the fit of cosmological parameters to the Wilkinson Microwave Anisotropy Probe (WMAP) data (Bennett *et al.*, 2005), have given support to a novel scenario for our universe. The observable universe, which may contain three density components, could in fact have serious departures from the previously assumed standard cosmological model. In this novel scenario, a dark energy dominates the universe today and drives its acceleration. This energy must be distributed smoothly on large scales, and be of negligible effect during early epochs. However, the amount of dark energy may in fact be of the same order of magnitude as the matter during a long period of cosmological

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history. In theory, this problem can be resolved by making modifications to the right-hand side of Einstein's field equation, but may require a fine tuning of the different density components of the universe. For example, an additional scalar field of matter may demand a tracking behavior in playing the role of dark energy or dark matter. Therefore, in this paper, we will attempt to make modifications to the left-hand side of Einstein's equations by adding geometry terms.

A number of models for the dark energy have been suggested. The model based on general relativity with a constant vacuum energy is by far the simplest one. This is effectively the same as the cosmological constant in standard general relativistic cosmology. However, the presently predicted values from theory are much greater than those inferred from observations, it is so big that we may have to appeal to the anthropic principle (Garriga and Vilenkin, 2000; Weiberg, 1987). The quintessence model, which invokes a very light and slowly evolving scalar field, requires its potential to be so flat that it is difficult to explain how the tiny mass of this field can stay safe from quantum corrections. Other models include a network of topological defects, and calls for extra dimensions. But all these have conceptual problems which need to be further clarified (Gutperle *et al.*, 2003; Kachru *et al.*, 2005; Kallosh *et al.*, 2000; Peebles and Ratra, 2005).

On the other hand, all searches for the signs of new physics beyond the particle standard model have only confirmed the remarkable success of the standard model. These confirmations have been attributed to the success of the Higgs mechanism, but the corresponding Higgs particle has not been found. May be we should change the concept. As we know, the Higgs mechanism requires that the Higgs scalar is coupled to all fundamental particles and provides their mass. Therefore, it should be a universal coupling, and possibly only gravitational interactions could do this. The coupling between Higgs' complex scalar and the electromagnetic gauge field should be of particular note, and for this coupling is also required in the standard process of Higgs mechanism. As far as it is known, there are only two kinds of interactions act on the photon field: the electromagnetic interaction and the gravitational interaction. Therefore, if it is assumed that Higgs' complex scalar is without charge of electricity, then the Higgs scalar can only be interpreted as gravitational. The transfer of the gravitational interaction may be realized through just such a coupling. Furthermore, the particle standard model is still not able to give a full interpretation of the origin of the hierarchy between the weak scale and the unification scale. It may also imply that this problem is connected with the running of the energy scale of universe.

In this letter, we would like to discuss the peculiar property of scalar gravity. First, we give a review of the cosmological property of the Brans–Dicke's gravitational scalar field theory in Section 2, then we give out our scalar gravity model and discuss its Higgs mechanism in Section 3, and some conclusion remarks are given in the last section.

2. COSMOLOGICAL PROPERTY OF GRAVITATIONAL SCALAR FIELD

Brans–Dicke theory is an alternative relativistic theory of gravity (Brans, 2005; Brans and Dicke, 1961). Compared with general relativity, as well as the metric tensor of space-time which describes the geometry there is an auxiliary scalar field ϕ which also describes the gravity. The testing of Brans–Dicke theory using stellar distances, the CMB temperature and polarization anisotropy have been discussed in (Chen and Kamionkowski, 1999; Gaztanaga and Lobo, 2001). In this section, we want to demonstrate the dynamical property of the gravitational scalar in the case of Brans–Dicke cosmology.

Before applying the Brans–Dicke theory to cosmology, we start by writing the Robertson–Walker line element as

$$ds^2 = -dt^2 + a^2(t)\tilde{g}_{ij}dx^i\, dx^j, \tag{1}$$

where i, j run from 1 to 3, $a(t)$ is the scale of the non-compact three-dimensional space with constant curvature K. The action of Brans–Dicke theory with non-vanishing vacuum energy reads

$$I = \frac{1}{16\pi}\int d^4x\sqrt{-g}\left(\phi R + \omega g^{\mu\nu}\frac{\nabla_\mu\phi\nabla_\nu\phi}{\phi}\right) + \int d^4x\sqrt{-g^4}(\mathcal{L}_{\text{matter}} - \Lambda), \tag{2}$$

where R is the space-time curvature scalar, ϕ is an auxiliary gravitational scalar field, ω is a parameter of Brans–Dicke theory, and the vacuum energy density from spontaneous symmetry breaking in quantum field theory is denoted by Λ. Here, $\Lambda > 0$. Then the corresponding field equations are

$$\begin{aligned} R_{\mu\nu} - \frac{1}{2}g_{\mu\nu}R = &-\frac{8\pi}{\phi}T_{\mu\nu} - \frac{1}{\phi}(g_{\mu\nu}\phi^{;\alpha}{}_{;\alpha} - \phi_{;\mu;\nu}) - \frac{\omega}{\phi^2}\phi_{;\mu}\phi_{;\nu} \\ &+ \frac{1}{2}\frac{\omega}{\phi^2}g_{\mu\nu}\nabla_\sigma\phi\nabla^\sigma\phi - \frac{8\pi}{\phi}\Lambda g_{\mu\nu}, \end{aligned} \tag{3}$$

and the field equation for ϕ reads

$$\Box^2\phi = \phi^{;\mu}{}_{;\mu} = \frac{8\pi}{-3+2\omega}(T_{\text{matter}} + 4\Lambda). \tag{4}$$

The matter stress–energy–momentum tensor may be written as $T_{\mu\nu} = (\rho_m + p_m)u_\mu u_\nu + p_m g_{\mu\nu}$, and then the classically conserved perfect fluid energy momentum tensor is

$$\frac{\partial\rho_m}{\partial t} = -3\frac{\dot{a}}{a}(\rho_m + p_m). \tag{5}$$

After a straightforward calculation using Equation (1), we also obtain the non-zero components of the Ricci tensor

$$^{3+1}R_{00} = -3\frac{\ddot{a}}{a}; \tag{6}$$

$$^{3+1}R_{ij} = \left(\frac{\ddot{a}}{a} + 2\frac{\dot{a}^2}{a^2} + 2\frac{K}{a^2}\right) g_{ij}; \tag{7}$$

Therefore, the fundamental equations of Brans–Dicke cosmology are

$$1 + \frac{K}{\dot{a}^2} = \frac{8\pi}{3H^2}\left[\frac{\rho}{\phi} - \frac{\omega}{16\pi}\frac{\dot{\phi}^2}{\phi^2} - \left(\frac{3H}{8\pi}\frac{\dot{\phi}}{\phi}\right) + \frac{\Lambda}{\phi}\right]; \tag{8}$$

$$\frac{\ddot{a}}{a} = -\frac{4\pi}{3}\left[\frac{(-6+2\omega)\rho + 6\omega p}{\phi(-3+2\omega)} - 4\left(\frac{\omega}{16\pi}\frac{\dot{\phi}^2}{\phi^2}\right)\right.$$
$$\left. -2\left(\frac{3H}{8\pi}\frac{\dot{\phi}}{\phi}\right) + \frac{-6-4\omega}{-3+2\omega}\left(\frac{\Lambda}{\phi}\right)\right]. \tag{9}$$

As was discussed earlier, if we here assume that the vacuum energy plays the role of dark energy in the present universe, therefore

$$\Omega_{\text{vacuum}} = \frac{8\pi}{3H^2}\left(\frac{\Lambda}{\phi}\right), \tag{10}$$

which is no longer a constant for a fixed Hubble parameter H. There would be an obvious depression in Ω_{vacuum} if the scalar field ϕ of Brans–Dicke theory rolls to a large number. In addition, the dynamical equations of ϕ can be derived from Equation (4), and are

$$\frac{\ddot{\phi}}{\phi} = -\frac{8\pi(\rho - 3p)}{\phi(-3+2\omega)} + 8\pi\left(-\frac{3H}{8\pi}\frac{\dot{\phi}}{\phi}\right) - \frac{32\pi}{-3+2\omega}\left(\frac{\Lambda}{\phi}\right). \tag{11}$$

As a remarkable character, it is urgently necessary to note that the Equation (11) contains almost all of cosmological density components demonstrated by Friedman's Equations (8) and (9).

3. SCALAR GRAVITY AND HIGGS MECHANISM

As a generalization of Brans–Dicke theory, we propose a new theory of gravity with a complex scalar field φ which is coupled to the curvature scalar. It is natural to construct the action as

$$I = \frac{1}{16\pi}\int d^4x\sqrt{-g}[\kappa\varphi\varphi^* R(t,x) + \omega g^{\mu\nu} D_\mu\varphi(D_\nu\varphi)^* - \lambda(\varphi\varphi^*)^2]$$
$$+ \int d^4x\sqrt{-g}\mathcal{L}_{\text{matter}}. \tag{12}$$

Here the coupling constants κ, ω and λ are all dimensionless, so it is consistent with the requirement of renormalizability. In addition, we also restrict them to be positive in our model.

In principle, the curvature scalar can always be decomposed into

$$R(t, x) = \bar{R}(t) + \tilde{R}(t, x), \tag{13}$$

here $\bar{R}(t)$ can be regarded as the average value of the scalar curvature: $\bar{R}(t) = \langle R(t, x)\rangle$ and $\tilde{R}(t, x)$ is the local perturbation induced by the actual matter distributing. In addition, it is natural to be assumed that in cosmology the homogeneous isotropic component $\bar{R}(t)$ would dominate in the dynamical equation of the universe. As far as the RW metric is concerned, this component can be written as

$$\bar{R}(t) = 6\frac{\ddot{a}}{a} + 6\frac{\dot{a}^2}{a^2} + 6\frac{K}{a^2}. \tag{14}$$

It is clear that the sign of the cosmological curvature scalar $\bar{R}(t)$ could change with the evolution of our universe theoretically. On the other hand, the potential of complex scalar φ in our case can be written as

$$V(\varphi) = -\kappa R(t, x)\varphi\varphi^* + \lambda(\varphi\varphi^*)^2. \tag{15}$$

The definition of physical vacuum may still require the space homogeneous and isotropic. Therefore, the actual vacuum should be considered according to the following formula: $\langle V\rangle(\varphi) = -\kappa\bar{R}(t)\varphi\varphi^* + \lambda(\varphi\varphi^*)^2$. When $\bar{R}(t)$ evolves into a positive quantity, there exist non-trivial minimums. The value of these minimums are distributed on the circle

$$|\varphi| = \sqrt{\frac{\kappa\bar{R}(t)}{2\lambda}} := \frac{v}{\sqrt{2}}. \tag{16}$$

Hence, the gauge symmetry breaks spontaneously. For convenience, we only consider the $U(1)$ gauge symmetry in this Letter. The Higgs mechanism requires the special gauge transformation

$$\varphi(x) \longrightarrow \varphi'(x) = \eta(x) + \frac{v}{\sqrt{2}}; \tag{17}$$

$$A_\mu \longrightarrow B_\mu = A_\mu + \frac{1}{e}\nabla_\mu\xi(x); \tag{18}$$

$$D_\mu = \nabla_\mu + ieA_\mu \longrightarrow D'_\mu = \nabla_\mu + ieB_\mu. \tag{19}$$

After this transformation the Equation (12) becomes,

$$I = \frac{1}{16\pi}\int d^4x\sqrt{-g}\left[\kappa\left(\eta + \sqrt{\frac{\kappa\bar{R}(t)}{2\lambda}}\right)^2 \tilde{R}(t, x) + \omega\nabla_\mu\left(\eta + \sqrt{\frac{\kappa\bar{R}(t)}{2\lambda}}\right)\right.$$

$$\times \nabla^{\mu}\left(\eta+\sqrt{\frac{\kappa \bar{R}(t)}{2\lambda}}\right)+\eta^{2}\kappa\bar{R}(t)-3\eta^{2}\kappa\bar{R}(t)-2\sqrt{2\kappa\bar{R}(t)\lambda}\eta^{3}-\lambda\eta^{4}$$

$$+\omega e^{2}B_{\mu}B^{\mu}\left(\eta+\sqrt{\frac{2\kappa\bar{R}(t)}{\lambda}}\right)\eta+\omega\frac{\kappa\bar{R}(t)}{2\lambda}e^{2}B_{\mu}B^{\mu}+\frac{\kappa^{2}\bar{R}^{2}(t)}{4\lambda}\Bigg]$$

$$+\int d^{4}x\sqrt{-g}[\bar{\mathcal{L}}_{\mathrm{matter}}(t)+\tilde{\mathcal{L}}_{\mathrm{matter}}(t,x)]. \tag{20}$$

In our framework, the running of the vacuum energy density with the evolution of the universe is realized. We have

$$\Lambda=\frac{1}{16\pi}\frac{\kappa^{2}\bar{R}^{2}(t)}{4\lambda}. \tag{21}$$

Here the Higgs field is a real gravitational scalar η, which is generated by a spontaneous symmetry breaking of the complex field φ. Therefore, the dynamical mass of the Higgs field is $m_{\eta}=\sqrt{\frac{2\kappa\bar{R}(t)}{\omega}}$ and the dynamical equation for the Higgs field is

$$\Box^{2}\left(\eta+\sqrt{\frac{\kappa\bar{R}(t)}{2\lambda}}\right)=\frac{1}{2\omega}\Bigg[-4\kappa\bar{R}(t)\eta-6\sqrt{2\kappa\bar{R}(t)\lambda}\eta^{2}-4\lambda\eta^{3}+2\omega e^{2}B_{\mu}B^{\mu}\eta$$

$$+\omega e^{2}B_{\mu}B^{\mu}\sqrt{\frac{2\kappa\bar{R}(t)}{\lambda}}\Bigg]+\frac{\kappa}{\omega}\left(\eta+\sqrt{\frac{\kappa\bar{R}(t)}{2\lambda}}\right)\tilde{R}(t,x). \tag{22}$$

We can imitate the discussion in Brans–Dicke theory and make a rough estimate of the average of $\eta+\sqrt{\frac{\kappa\bar{R}(t)}{2\lambda}}$ by computing the central potential of a gas sphere with the cosmic mass density $\bar{\rho}\sim 10^{-29}g\,\mathrm{cm}^{-3}$ and radius equal to the apparent radius of the universe $r\sim 10^{28}$ cm, this gives an average value

$$\left\langle\eta+\sqrt{\frac{\kappa\bar{R}(t)}{2\lambda}}\right\rangle\sim\frac{-4\kappa}{2\omega}\left(\eta+\sqrt{\frac{\kappa\bar{R}(t)}{2\lambda}}\right)\bar{R}(t)r^{2}$$

$$=\frac{-2}{\omega\left(\eta+\sqrt{\frac{\kappa\bar{R}(t)}{2\lambda}}\right)}\kappa\left(\eta+\sqrt{\frac{\kappa\bar{R}(t)}{2\lambda}}\right)^{2}\bar{R}(t)r^{2}$$

$$\sim \frac{-2}{\omega\left(\eta+\sqrt{\frac{\kappa\bar{R}(t)}{2\lambda}}\right)}16\pi\bar{\rho}r^2$$

$$\simeq \frac{-2}{\omega\left\langle\eta+\sqrt{\frac{\kappa\bar{R}(t)}{2\lambda}}\right\rangle}16\pi\times 10^{27}g\ \mathrm{cm}^{-1}. \tag{23}$$

Note that the constant $1/G = 1.35\times 10^{28} g\,\mathrm{cm}^{-1}$; hence, we normalize $\eta+\sqrt{\frac{\kappa\bar{R}(t)}{2\lambda}}$ so that

$$\omega\left\langle\eta+\sqrt{\frac{\kappa\bar{R}(t)}{2\lambda}}\right\rangle^2 \simeq \frac{1}{G}. \tag{24}$$

It is clear that the constant κ is still possible to be maintained in the magnitude order of $\mathcal{O}(1)$ in its post Newtonian formalism.

In fact, It is not only the gauge boson obtains mass (see Equation (20) for this boson mass). If it is assumed that the coupling between fermions and this gravitational scalar φ exists, the fermions can also obtain mass in this picture. Here we consider the simple Higgs-lepton coupling

$$G_{\mathrm{e}}\left[\overline{e_R}\varphi^+\begin{pmatrix}\nu_e\\ e\end{pmatrix}_L+(\overline{\nu_e}\quad \overline{e})_L\varphi e_R\right]. \tag{25}$$

After the gauge symmetry breaks, the electron obtains mass

$$m_e = G_{\mathrm{e}}\sqrt{\frac{\kappa\bar{R}(t)}{2\lambda}}. \tag{26}$$

Therefore, the mass of fundamental particles in this scenario depend on the cosmological curvature scalar at spontaneous symmetry breaking, and are not uniquely fixed quantities any longer. However, as we discussed earlier, whether the gauge symmetry breaks or not is determined by the sign of the curvature scalar, and is also determined by the evolving energy scale of the universe. Hence, fundamental particles could not be distributed homogeneously on all physical energy scales in the present time. In addition, experiments have also shown that the mass of fundamental particles are stable at the present time. We think mass stability may be obtained by considering the average value of the cosmological scalar curvature $R(t,x)$, just in a similar way to which Newton's constant G can be related to the average value of the scalar field ϕ in Brans–Dicke theory.

According to Equation (26), it can also be investigated that the running speed of the electron mass in our case is given by

$$\dot{m}_{\rm e} = G_{\rm e}\frac{1}{2}\left(\sqrt{\frac{\kappa\bar{R}(t)}{2\lambda}}\right)^{-1}\frac{\kappa\dot{\bar{R}}(t)}{2\lambda}. \tag{27}$$

We recall the expression (14) of the curvature scalar in the homogeneous isotropic universe, and further regard that $\dot{a}^2/a^2$ is still in the same magnitude order with $\ddot{a}/a$ in the present scenario, it is natural to be extrapolated that $\bar{R}(t) \sim \mathcal{O}(H^2)$ and $\dot{\bar{R}}(t) \sim \mathcal{O}(H^3)$. Hence, we can also estimate the observational effect of a variable electron mass in the present time,

$$\frac{\Delta m_{\rm e}/m_{\rm e}}{\Delta t} \simeq \frac{\dot{m}_{\rm e}}{m_{\rm e}} = \frac{1}{2}\frac{\dot{\bar{R}}(t)}{\bar{R}(t)} \sim \mathcal{O}(H) \sim 10^{-17}\ {\rm s}^{-1}. \tag{28}$$

On the basis of the cosmological principle, the distribution of the ordinary matter is assumed to be homogeneous isotropic. Then the motion of the ordinary matter on the cosmological scale is also homogeneous isotropic. Therefore, on the cosmological scale, the gravity theory may approximately have the formula of

$$\begin{aligned} I_{\cos} = \frac{1}{16\pi}\int d^4x\sqrt{-g}\Bigg[&\omega\nabla_\mu\left(\eta+\sqrt{\frac{\kappa\bar{R}(t)}{2\lambda}}\right)\nabla^\mu\left(\eta+\sqrt{\frac{\kappa\bar{R}(t)}{2\lambda}}\right)+\eta^2\kappa\bar{R}(t)\\ &-3\eta^2\kappa\bar{R}(t)-2\sqrt{2\kappa\bar{R}(t)\lambda}\,\eta^3-\lambda\eta^4+\omega e^2B_\mu B^\mu\left(\eta+\sqrt{\frac{2\kappa\bar{R}(t)}{\lambda}}\right)\eta\\ &+\omega\frac{\kappa\bar{R}(t)}{2\lambda}e^2B_\mu B^\mu+\frac{\kappa^2\bar{R}^2(t)}{4\lambda}\Bigg]+\int d^4x\sqrt{-g}\bar{\mathcal{L}}_{\rm matter}(t). \end{aligned} \tag{29}$$

The motion of cosmological scale can be ignored in present tests of a gravity theory in the solar system. Hence, in contrary to the cosmology on the large scale, the gravity theory on the solar scale as a local perturbation formula may be approximately taken as

$$I_{\rm sol} = \frac{1}{16\pi}\int d^4x\sqrt{-g}\kappa\left(\eta+\sqrt{\frac{\kappa\bar{R}(t)}{2\lambda}}\right)^2\tilde{R}(t,x)+\int d^4x\sqrt{-g}\tilde{\mathcal{L}}_{\rm matter}(t,x). \tag{30}$$

4. CONCLUSION REMARKS

In this Letter, we have proposed a new form of scalar gravity theory, in which the gravitational complex scalar naturally provides a candidate Higgs field for Higgs mechanism. To demonstrate the dynamical property of such a gravitational scalar in cosmology. We have also investigated the fundamental equations of Brans–Dicke cosmology as an analog. In our scenario, the vacuum energy density is required to be running with the evolution of the universe, which may release the particle standard model from the rigorous cosmological constant problem in some sense. Besides, a present testing-survivable running of the mass scale of fundamental particles is also realized in our model and may shed light on the hierarchy problem. As far as the spirit of this letter is concerned, there are two outcomes may deserve to be emphasized in this conclusion. One of the key ideas is that if a spontaneous symmetry breaking of the coupling between the curvature scalar and a gravitational scalar field occurs on the cosmological scale, some additional geometrical terms can be naturally introduced into the field equations. Secondly, we have argued that a gravitational scalar is also qualified to be a candidate for the Higgs field. Our present model is however still simplistic and will be clarified further in our forthcoming papers.

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